

# Self -guided learning in maths - scenarios, material from a German pilot project

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## Abstract:

How can new media make the teaching and learning of mathematics more exciting? This is one of the questions that the project 'SelMa - Selbstlernen in der gymnasialen Oberstufe – Mathematik', tries to answer. SelMa is supported by the Federal Republic of Germany and the state of North Rhine-Westphalia. Different scenarios of self-guided learning have been created. Some of them and results of the first evaluations are presented.

## 1. At a glance - organization of the pilot project

The four-year pilot project "Self guided learning in mathematics in senior high schools" - called **SelMa**- started in the beginning of 1999. It aims to analyse the interdependencies and interactions between mathematics, learning in general and the use of new media.

Main questions focussing upon learning, mathematics and use of media are e.g.:

- What are mathematical topics, that match the idea of self-guided learning?
- How should classroom material be arranged and presented? Online or offline?
- How do students and teachers have to be supported?
- How can progress in learning be 'controlled'?
- How can new media improve the quality of learning (mathematics) and provide life-long learning?
- How can knowledge be consolidated by means of intelligent practice?
- How can telecommunication (platforms for collaborative learning or a "teacher-on-demand via email) support learning of mathematics?

A team of 3-4 teachers of 5 schools in North-Rhine-Westfalia, called 'authors', began - addressing the issues mentioned above - to create scenarios of self-guided learning and develop exemplarily suitable classroom material.

In 2000, when the first projects were finished and successfully tested by the authors in their own classes, a second group of 10 schools, so-called 'evaluators', was established to evaluate the material and to systematically try out whether it works in everyday usage. Other schools have been invited to participate, too. The feedback of all evaluators will be incorporated in the on-going development of the material. The current state of affairs can be viewed on-line (<http://www.learn-line.nrw.de/angebote/selma/index.htm>). This site offers a wide range of material for trial or individual adaptation. The web-site has been created to be a platform for information, communication und co-operation between teachers in the field of self-guided learning of mathematics.

Furthermore, authors and evaluators are going to disseminate their practice in in-service-teacher-training, in order to built up networks of schools in the different regions. By this, we hope, that periods of self-guided learning will occur more often in an increasing number of schools. Publishers are to be included at an early stage. This is expected to lead to high-quality (offline and online) media that support most of the periods of self-guided learning of mathematics.

## **2. Self-guided learning : different scenarios and concepts**

The first projects of the authors include very different concepts of self-guided learning in math classes. Up to now there has been a deficiency of concrete concepts and learning arrangements. Documentations in German educational research of mathematics, that are detailed enough to transfer these ideas to other mathematical topics, are rather rare.

In one scenario of self-guided learning learning environments are intensively used. Students work on it in longer periods in math lessons and at home. The material is developed as a hypertext with exercises, contextual aid, a glossary, solutions and general advice to optimize individual and collaborative learning in school and at home. Another group of authors has established an independent learning centre (for all subjects) at their school. Some subjects of our math curriculum are exposed to self-guided learning, that means that these topics are not taught collectively in math lessons but exclusively learned by the students on their own without any support of the teacher. These learning environments consist of a course - rather linear structured - with graded aid, suggested solutions and a collection of problems - real-life problems of different categories.

Another different way to increase student activity and self-guided learning is the method of "working at stations". Ten to twenty different stations (exercises, real-life problems depending on the subject) are offered to the students. Some stations deal with a special task, a new mathematical context, others invite to exploration or investigation using handheld computers. All stations offer special aids with initial approaches and other tips suitable to the student's need and a paper with a complete solution. Each student receives a 'to-do-list', which informs about all the stations (number; title; topic; obligatory or additional station; individual, pair or group work, media). Students can choose the order of tasks and might individually (or in groups) choose their learning pace.

These two methods imply certain dangers. During periods of self-guided learning teachers automatically changes their role, from being an instructor to a supporter of individual learning processes. Normally teachers cannot exactly point out if and how big the learning progress of the groups in general and the individual really is. Students must be capable of monitoring their learning process on their own, but they must have learned this as well as their mathematical topics. Additional methods like learning diaries, mindmaps and electronical communication tools might support this process of self-control.

## **3. Scenarios and material: use and its evaluation in the classroom**

### **3.1 Learning environments**

Educational research tells us that learning and understanding of mathematical concepts and using problem solving strategies work better if there are various initial approaches with real

life problems of various levels matching different interests of the learners. As the understanding of math requires the knowledge of details, conclusions and relations between single topics, learning environments (based on hypermedia in a linked-up, but not linear structure) are advantageous to construct a network of mathematical knowledge in the learner's mind.

The pilot study 'SelMa' offers examples of a learning environment, e.g. 'linear programming/optimization' for coordinate geometry and another one for 'matrices'.

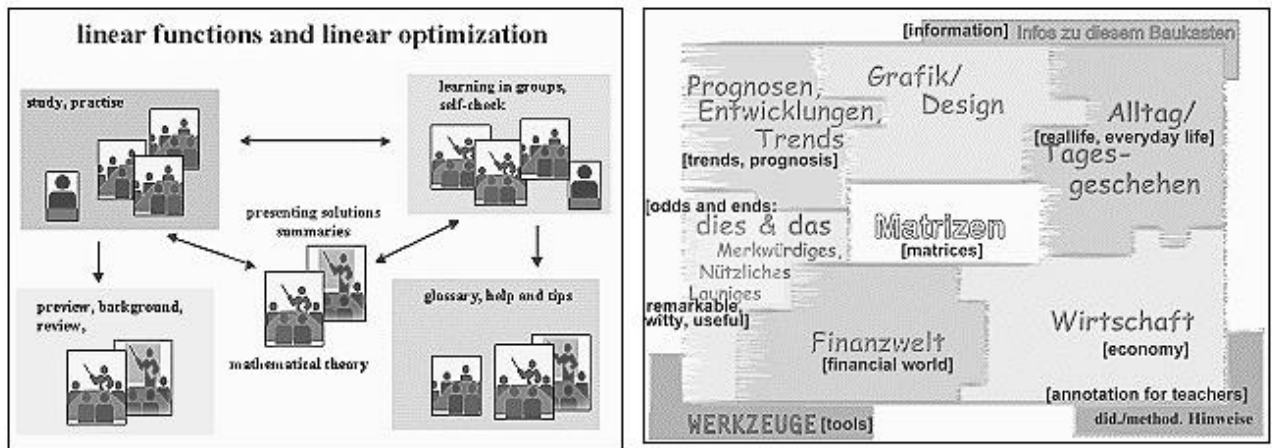
This learning environment is developed to be a revision of concepts around linear functions. The students choose one briefly described problems of a collection being the 'heart' of the learning environment, are guided to the important steps to solve a mathematical optimization problem. At the same time they revise working with linear functions they were taught at lower secondary level. The learning unit links new contents and mathematical concepts with topics that the students learnt (and have already forgotten) in previous mathematics lessons. One part of this learning environment deals with the learning process and his monitoring by the students themselves. Here students find e.g. advice for self-control and tips to optimize group work and study at home.

In the learning environment 'matrices' the students are offered several problems of different parts of real-life problems leading to the same mathematical topic of the learning subject 'matrices'. They have to choose one problem that they are most interested in, they are 'guided' through the rather open problem - not step by step - but by more general questions concerning the way of problem solving, by a glossary or by questions that prepare building up the mathematical theory behind the problem. New definitions and theorems e.g. will be found whole class-working. Students can see that different problems lead to the same mathematical concepts.

Both learning environments that are intended to be used in classroom and at home offer some more details, that support orientation and self-guided learning in hypermedia:

- survey of the learning subject, table of contents, glossary, review of the topic
- some (recommended) provided paths
- different modes of representation, visualization and interactions as often as possible
- some interesting historical facts concerning the topic, real-life applications
- exercises and contextual, graded aid
- a chapter concerning learning strategies, problem solving, self control of the learning process.

The hypertext based on HTML is open, can easily be modified (e.g. integration of other documents and visualizations- either static or dynamic, interactive and ready to be published in the internet), improved, developed, discussed ... The material includes practical advice for the teacher, who becomes an *individual adviser* during the work with this learning environment and will become a *moderator of the class*, when the results of the first groupwork will be discussed in class and general procedures of solving optimization problems will be found.



**Fig. left:** different parts and their role within the learning environment and important activities without the learning environment;

**Fig. right:** main page of the learning environment 'matrices'

### 3.2 Learning at Stations

The project "Geometry of Circles" consists of two parts. In the first part the students have to investigate the equation of a circle and then create - by use of CAS or graphing calculator - a mathematical description of an existing company logo, a window of a church, a pattern or a model of an existing object containing different circles. Here, students can see the importance of geometry in real life. The students work in groups of two or three and have to present their results on posters or word-documents to all students.

The second part of the project is based on the method 'learning at stations', often practised in elementary schools. It focusses on the development of new aspects of coordinate geometry, problems connecting the new geometrical object circle with other known objects like parabola and lines (tangent, points of intersection, ...).



**Fig. 3:** Students working in groups at the same station or using CAS for exploration

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<b>"To-Do-List" Circles</b>				
<b>This is your own to-do-list. Please remark the station you had finished and make annotations of any question!</b>				
Nr.	topic	important to know	how difficult?	o.k.?
1	finding equations of circles	revision	✓	
2	intersection line-circle		✓✓	
3	tangent line		✓✓	
4	points of intersection of 2 circles		✓✓	
5	does every equation fit to a circle?		✓	
6	family of circles	additional	✓✓✓	
7	puzzle of circles, equations and descriptions	revision/self-check	✓	
8	circles and lines	 Nr. 2	✓✓	
9	more complex exercise	last station?	✓✓✓ ✓✓✓	
10	equation of a tangent line, that reminds of the position of the circle	Nr. 2 before	✓✓✓	

**Fig. 4:** "To-Do-List" of working at stations concerning 'circles'

The problems are presented on worksheets and files, first with the help of concrete exercises, then by generalizing the solution. Each station consists of the worksheet, some helpful questions and a complete solution to check. Some stations are more graphically oriented, e.g. investigations of family of curves or a circle-puzzle where descriptions, graphs and equations of circles have to fit correctly.

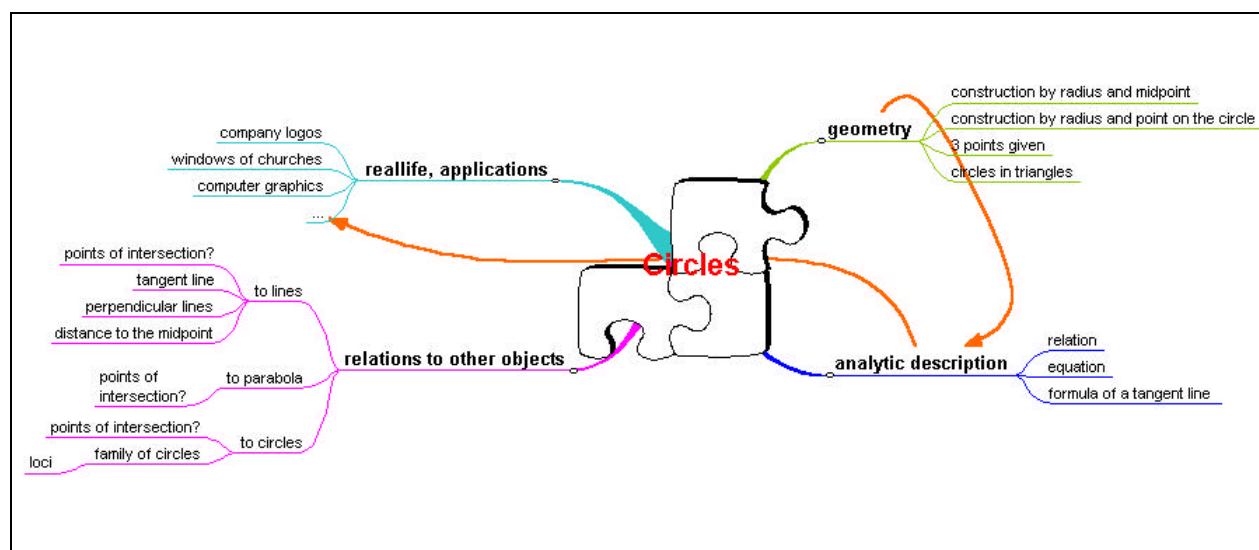
In this project, individual learning is particularly supported by having pupils study materials at different learning stations. The complete offer consists of 11 stations. The stations are set up in such a way that different background levels of learning, different speeds of learning and working, and different needs of working (individually, in groups of two or three, or alone) are accommodated.

Different media are used at different stations, e.g. the CAS DERIVE or the TI-89 calculator, but also a puzzle to revise topics of the first part of the project. The tasks are usually activity-oriented. The students normally work in groups of two or three and decide together at which station to work next. The material of all stations is laying on a table during the lesson. Each station exists 3-4 times, so that the students can really choose what to do next. During the

work the teacher answers questions of the groups. We noticed in our first evaluation that students only tentatively used the additional aid, put on a table further away from the exercises. First they tried to help each other, then they asked the teacher, who had much more time to give individual advice than in traditional lessons. Collaborative working is highly supported by this method. No students checked the problem using the solution provided before having tried to solve it on their her own.

### 3.3 Mindmaps, diaries and communication tools

After longer periods of self-guided learning weaker students sometimes do not know whether they have learned all topics and understood all relations between new and old subjects. Mindmaps can support the review about the main steps of the learning process in different ways.



**Fig.4:** Mindmap of the learning subject "circles"  
(translated in English, made with the help of the software 'Mindmanager')

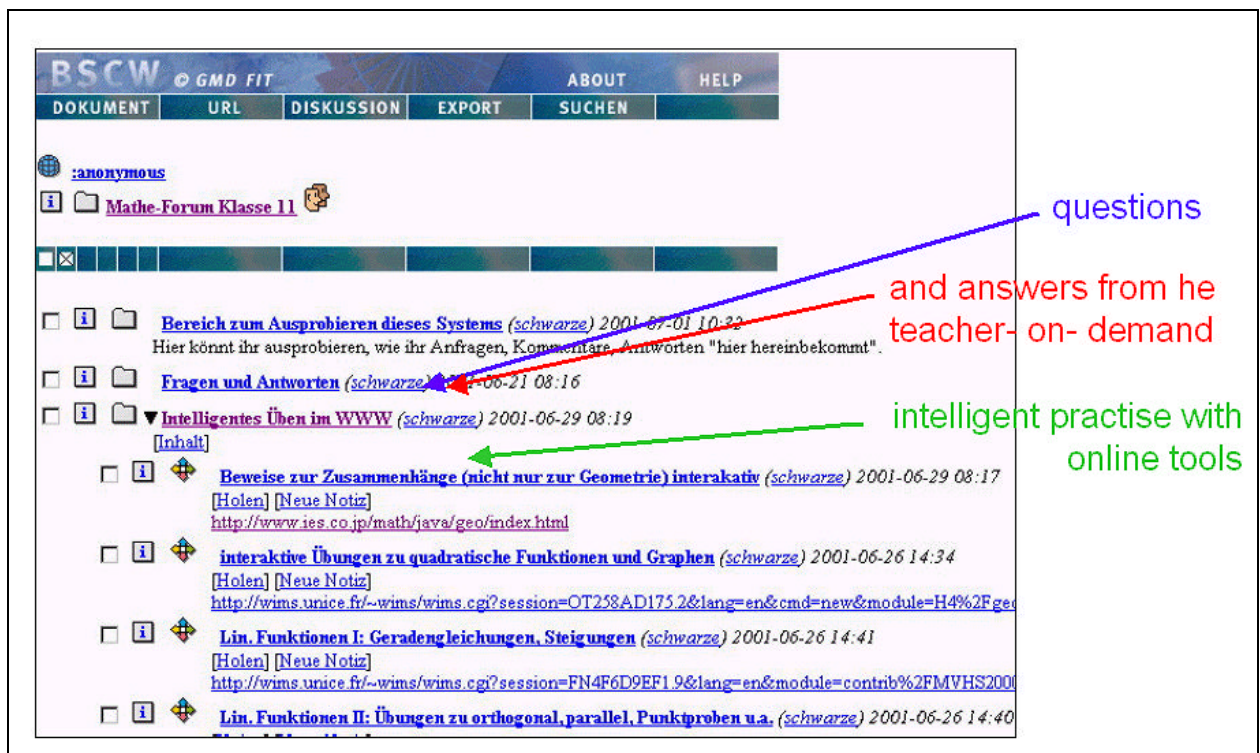
First, a mindmap containing only main topics can be filled in individually completed after the period of self-guided learning. So the individual automatically reflects his own learning-progress. Newly gained facts are linked to details of the 'old' individual network of knowledge. Different mindmaps - that means different points of view- can be presented and discussed in class.

Secondly, a mindmap of the subject matter can be constructed whole-class learning. Mindmapping tools like 'Mindmanger' or 'Inspiration' offer various features like e.g. annotations, links and moving of branches to other positions. Students have intensively to discuss links between different branches and will become aware of structure, connections and relations between mathematical topics of the subject matter.

Diaries are a mechanism, that leads students to reflect their progress of learning continuously. In an introductory session students of Year 11 were informed of the aims of this method and the contents of the personal diary. A personal diary - only read by the writer and the teacher -

should briefly contain all important facts of a lesson (*steps* to a new topic, definitions, proofs, examples) and may include a personal review (What have I learned? What was difficult for me to understand? How do I memorize it?). The diaries were checked and annotated with sticky notes and assessed by the teacher in a three-month-period. Most of the students who kept a diary containing a lot of personal annotations stated that they felt better prepared for tests because they had paid more attention to their weaknesses compared to the beginning of the school year. The SelMa-web-site presents and documents 25 exemplars from diaries in the six fields of reflections on lessons, aha-effects, individual explanations, self-assessment, analyses of mistakes, and further issues.

In addition we started to experience with collaborative online tools like BSCW or Web-CT that can be integrated in longer periods of self guided learning. All student have access to the internet at school, most of them at home, too.



**Fig.5 :** BSCW-Workspace (BSCW= basic support for co-operative work); see <http://bscw.gmd.de>

We invite students of Year 11 to use a workspace in the internet as an additional offer. They may put individual questions concerning the learned topics of the last lessons, they get corresponding answers from the 'teacher on demand' or from other students. Weaker students can find intelligent practice, links to interactive online-tools and visualizations, brighter students may be interested in further (anspruchsvollere) tasks and experiments e.g. with CAS. The tools mentioned above encourage the exchange of information between the students and initiate discussions with experts. 'Communicating math' seems to be more intensive when using these new technologies.

#### 4. References and websites

- BSCW-website: <http://bscw.gmd.de>
- BSCW-workspace with public access: <http://bscw.gmd.de/pub/german.cgi/0/27877615>
- Fankhänel, Weber: SelMa – New Perspectives for Self-guided Learning in Teaching Mathematics at Senior High School Level. Paper for WCCE, 2001, Kopenhagen.
- Inspiration (software): <http://www.inspiration.com>
- Mindmanager: <http://www.mindjet.com>
- SelMa-website: <http://www.learn-line.nrw.de/angebote/selma/>
- project "matrices" :  
[http://www.ham.nw.schule.de/beisenkamp/selma/matrix1/HTML-Dateien/Matrix\\_Selma/](http://www.ham.nw.schule.de/beisenkamp/selma/matrix1/HTML-Dateien/Matrix_Selma/)
- project "linear optimization":  
[http://www.learn-line.nrw.de/angebote/selma/foyer/02b\\_hammproj1.htm](http://www.learn-line.nrw.de/angebote/selma/foyer/02b_hammproj1.htm)
- project "geometry of circles":  
<http://www.ham.nw.schule.de/projekte/swmathe/selma/kreis2001/>
- Personal web-site: <http://www.mathematikunterricht.de/beisenkamp/>
- PP-presentation of the talk: <http://mathematikunterricht.de/Tagung/ictmt5/index.htm>